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A Veneziano model, with absorptive corrections, for higher spin production

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Abstract. A Veneziano model is applied to higher spin production processes in the near forward scattering region over an extended energy range, for the processes $K^+p \to K^0\Delta^{++}$ and $K^+p \to K^{*+}p$. Absorption corrections are included to take some account of unitarity, and the U(6, 6) symmetry scheme is used to determine the coupling constants. While the results for $K^{*+}p$ are an improvement on previous Regge calculations, the results for $K^0\Delta^{++}$ are similar and suffer from the fact that the $\bar{N}\Delta$ coupling constant is too small.

1. Introduction

The duality between the Regge pole description, and resonance description of scattering amplitudes, proposed first by Dolen *et al* (1968) has led to the Veneziano model (Veneziano 1968). The Veneziano formula is crossing symmetric, reduces to the Regge pole formula at high energies, and can be expanded as a sum of direct-channel poles of zero width. Thus it contains all the dynamical information one needs, if one believes that only Regge poles are important in the high energy description of scattering amplitudes.

However, it is well known that cuts are required to obviate the necessity for the introduction of fictitious particles, notably the ρ' and the pion conspirator. The traditional way of including cuts is by means of absorption corrections (Gottfried and Jackson 1964). This has the effect of taking some account of unitarity, the violation of which is one of the more outstanding problems connected with the Veneziano model.

The invariant amplitudes of a process contain all the dynamical content of the description of the scattering. In addition they are assumed to be free of kinematic singularities. Therefore, if one wishes to construct a Veneziano model for a particular process, it is the invariant amplitudes which one parametrizes with a Veneziano formula. A number of attempts have been made to construct models for $0^- + \frac{1}{2}^+ \to 0^- + \frac{1}{2}^+$ processes (eg Adjei *et al* 1972, Berger and Fox 1969 and Lovelace 1969). In view of the success of these earlier applications it seems worthwhile to extend the technique to cases where the spin complications are greater. We shall consider the processes $K^+p \to K^0\Delta^{++}$ ($0^- + \frac{1}{2}^+ \to 0^- + \frac{3}{2}^+$), and $K^+p \to K^{*+}p$ ($0^- + \frac{1}{2}^+ \to 1^- + \frac{1}{2}^+$). Plotting the differential cross section only is not an adequate test of the inclusion of spin factors. A more satisfactory test is to plot the spin density matrices for the outgoing high spin particle. In this paper these are calculated in the Jackson frame.

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The predictions of the model for a range of energies are presented. The results compare favourably with the experimental data.

2. Formalism

Since the invariant amplitudes contain the dynamical description of the reactions, it is these which we chose to represent with a Veneziano formula.

The M functions for the two processes we wish to consider are (Jones and Scadron 1968):

(i) $K^+p \rightarrow K^0\Delta^{++}$

$$M_\mu = A(s,t)\gamma_5 P_\mu + B(s,t)\gamma_5 P'_\mu + C(s,t)\gamma_5 \not{P}' P_\mu + D(s,t)\gamma_5 \not{P}' P'_\mu$$

where A, B, C, D are functions which contain a total of four arbitrary coupling constants.

(ii) $K^+p \rightarrow K^{*+}p$

$$M_\mu = A'(s,t)\gamma_5 P_\mu + B'(s,t)\gamma_5 P'_\mu + C'(s,t)\gamma_5 \not{P}' P_\mu + D'(s,t)\gamma_5 \not{P}' P'_\mu$$
$$+ E'(s,t)\gamma_5\gamma_\mu + F'(s,t)\gamma_5 \not{P}'\gamma_\mu$$

where A', B', C', D', E', F' contain a total of six arbitrary constants. $P = p_2 + p_4$ and $P' = p_1 + p_3$, where p_1 and p_3 are the four momenta of the incoming and outgoing mesons, and p_2 and p_4 the momenta of the incoming and outgoing fermions respectively.

A problem is that of the numerous coupling constants (particle–particle–reggeon) which may either be regarded as arbitrary parameters and fitted to the data, or may be inter-related, for example, by a higher symmetry scheme. We prefer to take the latter approach and use the U(6, 6) symmetry scheme (Delbourgo 1965a, Bég and Pais 1965, Sakita and Wali 1965 and Delbourgo *et al* 1965b) to reduce the number of arbitrary coupling constants. A Regge model, using the U(6, 6) symmetry and taking some account of unitarity by means of absorptive corrections, has been successful in fitting a great number of meson–baryon scattering processes (Adjei *et al* 1971a). We use this Regge model as a starting point for constructing our Veneziano model. This has already been done for $\bar{K}N$ and KN charge exchange reactions (Adjei *et al* 1972). We shall describe only briefly the construction of the Regge pole model as this is dealt with more fully in Adjei *et al* (1971a).

The T matrix for a Regge pole amplitude is constructed by first writing the one particle exchange term in the form

$$\langle p_3 p_4 | T | p_1 p_2 \rangle = \frac{\langle p_3 p_4 | J_5(1) J_5(2) | p_1 p_2 \rangle}{t - m^2}$$

for the pseudoscalar exchange, and

$$\langle p_3 p_4 | T | p_1 p_2 \rangle = \frac{\langle p_3 p_4 | J_\mu(1)(-g_{\mu\nu} + Q_\mu Q_\nu/m^2) J_\nu(2) | p_1 p_2 \rangle}{t - m^2}$$

for vector exchange, where the currents are obtained from Delbourgo *et al* (1956b) and where $Q = p_2 - p_4$. '1' and '2' refer to the meson and baryon vertices respectively. The amplitude is then reggeized as in Adjei *et al* (1971a). This results in the following form for the T matrix elements.

(i) $K^+p \to K^0\Delta^{++}$

Here ρ and A_2 exchanges are allowed. They are taken to be exchange degenerate as implied by duality. The T matrix is given by

$$X\epsilon_{\nu\kappa\lambda\rho}P'_\nu P_\kappa Q_\lambda \bar{D}_\rho N$$

where D_ρ is the decuplet field and N the nucleon field, and where†

$$X = \frac{3}{2}GHH_F g\frac{1}{2m_{24}^2}\left(1+\frac{2m}{\mu}\right)\Gamma(1-\alpha(t))\left(\frac{s}{s_0}\right)^{-(1-\alpha(t))}\alpha'$$

and

$$m_{24} = \frac{m_2+m_4}{2}.$$

m is the average mass of all the baryons in the octet and decuplet, and μ the average of the meson masses. The values of these masses are given in Adjei *et al* (1971a).

(ii) $K^+p \to K^{*+}p$

Here the possible exchanges are π, B, ρ, A_2, ω, ϕ, η. Because there are no trajectories coupling to the s channel duality implies that the π and B, the ρ and A_2, ω and f, ϕ and f′ and η and D are exchange degenerate.

(*a*) For pseudoscalar exchange, we get

$$T = X'\bar{N}\gamma_5 NP'\epsilon^*(p_3)$$

where

$$X' = \alpha' 3GHG_{D+2/3F-S}H_F\left(1+\frac{2m}{\mu}\right)\left(1-\frac{\mu_2^2}{4m^2}\right)\Gamma(-\alpha)\left(\frac{s}{s_0}\right)^{\alpha(t)}$$

(*b*) and for vector exchange

$$T = -Y\bar{N}N\epsilon_{\nu\kappa\lambda\rho}P_\nu\epsilon_\kappa^*(p_3)P'_\lambda Q_\rho - Z\epsilon_{\nu\kappa\lambda\rho}\bar{N}\gamma_\nu N\epsilon_\kappa^*(p_3)P_\lambda^1 Q_\rho$$

where

$$Y = 3GHH_D\left\{G_{F+3S}\left(1+\frac{\mu}{2m}\right)-\left(1+\frac{2m}{\mu}\right)G_{D+2/3F-S}\right\}\frac{1}{2m_2(m_1+m_3)}$$

and

$$Z = 3GHH_D G_{D+2/3F-S}\left(1+\frac{2m}{\mu}\right)\frac{(1-m^2/4m_2m_4)}{2(m_1+m_3)}.$$

G is the universal baryon–baryon–meson vertex coupling and H is the meson–meson–meson vertex coupling. These are the only undetermined parameters and these can be fitted from the Chew–Low extrapolation of the πNN coupling constant and the ρππ decay width respectively. $H_D, H_F, G_{D+2/3F-S}, G_{F+3S}$ are SU(3) Clebsch–Gordan coefficients which are exhibited in table 1 for the different poles which contribute to this process.

† G, H, H_F, g etc are couplings defined as in Adjei *et al* (1971a) and below.

Table 1. Clebsch–Gordan coefficients

(i) $K^+p \to K^0\Delta^{++}$

Pole	g	H_F
ρ	$-\sqrt{2}$	$\sqrt{2}$

(ii) $K^+p \to K^{*+}p$

Pole	$G_{D+2/3F-S}$	G_{F+3S}	H_D	H_F
π, A_2	$\frac{5}{3}$	1		1
ρ, B	$\frac{5}{3}$	1	1	
ω	$-\frac{1}{3}$	3	1	
ϕ	$-\frac{2}{3}\sqrt{2}$	0	$\sqrt{2}$	
η	$1/\sqrt{3}$	$\sqrt{3}$		$\sqrt{3}$

From these Regge amplitudes we need to extract the invariant amplitudes. We do this by using an equivalence relation (Jones and Scadron 1968), which is founded on the relation

$$\gamma_5\epsilon_{\alpha\beta\gamma\delta} = \gamma_\alpha\gamma_\beta\gamma_\gamma\gamma_\delta - g_{\alpha\beta}\gamma_\gamma\gamma_\delta - g_{\gamma\delta}\gamma_\alpha\gamma_\beta - g_{\alpha\delta}\gamma_\beta\gamma_\gamma - g_{\beta\gamma}\gamma_\alpha\gamma_\delta + g_{\alpha\gamma}\gamma_\beta\gamma_\delta + g_{\beta\delta}\gamma_\alpha\gamma_\gamma + g_{\alpha\beta}g_{\gamma\delta} - g_{\alpha\gamma}g_{\beta\delta} + g_{\alpha\delta}g_{\beta\gamma}.$$

This results in the decompositions:

(i) $K^+p \to K^0\Delta^{++}$

$$A = X\{E_4(E_1+E_3)+q^2+qK\cos\theta\}$$

$$B = X\left(\frac{m_4^2+m_2^2-t}{2}+m_4m_2\right)$$

$$C = Xm_4$$

$$D = 0$$

in the notation of Adjei *et al* (1971a).

(ii) $K^+p \to K^{*+}p$

(*a*) From spin O^- exchange $A = C = D = E = F = 0$
$B = X'$

(*b*) From 1^- exchange

$$A = (m_3^2-m_1^2)Y$$

$$B = 2\left(m_2^2+\frac{s-u-t}{2}\right)Y+(m_2+m_4)Z$$

$$C = (m_2+m_4)Y+Z$$

$$D = 0$$

$$E = (m_2+m_4)(s-u)Y-(s-u)Z$$

$$F = (t-4m_2^2)Y+(m_2+m_4)Z.$$

Now the invariant amplitudes may be expressed in a Veneziano form. This is done by replacing $\Gamma(1-\alpha(t))(s/s_0)^{\alpha(t)-1}$ by the simplest Veneziano term, chosen so that all the helicity amplitudes obey the constraint placed on them by the asymptotic behaviour of $d\sigma/dt$, that is, $\phi_i \simeq s^{\alpha_M}$ in the forward direction and $\phi_i \simeq s^{\alpha_B}$ in the backward direction where ϕ_i are helicity amplitudes (Berger and Fox 1969). The asymptotic behaviour of the invariant amplitudes is shown in table 2. As a first approximation only a single Veneziano term has been included.

Table 2. Asymptotic behaviour of invariant amplitudes

(i) $K^+p \rightarrow K^0\Delta^{++}$

Amplitude	Forward	Backward
A	$s^{\alpha(t)-1}$	$s^{\alpha_B(u)-5/2}$
B	$s^{\alpha(t)-1}$	$s^{\alpha_B(u)-5/2}$
C	$s^{\alpha(t)-1}$	$s^{\alpha_B(u)-3/2}$

(ii) $K^+p \rightarrow K^{*+}p$

Amplitude	Forward	Backward
A	$s^{\alpha(t)-1}$	$s^{\alpha_B(u)-3/2}$
B (P)	$s^{\alpha(t)}$	$s^{\alpha_B(u)-3/2}$
B (V)	$s^{\alpha(t)-1}$	$s^{\alpha_B(u)-5/2}$
C	$s^{\alpha(t)-1}$	$s^{\alpha_B(u)-3/2}$
E	$s^{\alpha(t)-1}$	$s^{\alpha_B(u)-3/2}$
F	$s^{\alpha(t)-1}$	$s^{\alpha_B(u)-5/2}$

where (P) stands for the pseudoscalar contribution and (V) for the vector.

A consequence of the linear trajectories used in the Veneziano model is that the fermion trajectories are parity doubled. Methods have been devised to remove the parity doublets for specific models (Adjei *et al* 1971b, Carlitz and Kislinger 1970), but these are difficult to implement. Lovelace (1969) and Amann (1970) have suggested that an infinite number of satellite terms can be added, and that these terms can be so adjusted as to remove the parity doublets on the leading trajectory, while still maintaining the same asymptotic behaviour. For the sake of simplicity, however, we choose to use only the simple Veneziano term.

In both processes, no s channel resonances can couple, so we need only the $V(t, u)$ term in the amplitude. In $K^+p \rightarrow K^0\Delta^{++}$ only the Y_1^* u channel pole can couple, so the Veneziano formula we require is

$$\frac{\Gamma(N-\alpha(t))\Gamma(M-\alpha_{Y'}(u))}{\Gamma(R-\alpha(t)-\alpha_{Y'}(u))}$$

where N, M and R are given for each invariant amplitude in table 3. For $K^+p \rightarrow K^{*+}p$ both the Y_0^* and Y_1^* trajectories can couple to the u channel, so we must write as our Veneziano term

$$(1-P)\frac{\Gamma(N-\alpha(t))\Gamma(M-\alpha_{Y_1}(u))}{\Gamma(R-\alpha(t)-\alpha_{Y_1}(u))}+P\frac{\Gamma(N-\alpha(t))\Gamma(M-\alpha_Y(u))}{\Gamma(R-\alpha(t)-\alpha_Y(u))}$$

which combination guarantees the correct asymptotic behaviour. P is a parameter to be fitted to the data. If we regard P as a measure of the relative strengths of the couplings of the Y_0^* and the Y_1^* trajectory to the u channel system, this will be the same in each invariant amplitude. Again, N, M and R are displayed in table 3.

Table 3.

Process	Amplitude	N	M	R
$K^+p \to K\Delta^{++}$	A	1	$\frac{5}{2}$	$\frac{7}{2}$
	B	1	$\frac{5}{2}$	$\frac{7}{2}$
	C	1	$\frac{3}{2}$	$\frac{5}{2}$
$K^+p \to K^{*+}p$	A	1	$\frac{3}{2}$	$\frac{5}{2}$
	B (P)	0	$\frac{3}{2}$	$\frac{5}{2}$
	B (V)	1	$\frac{5}{2}$	$\frac{7}{2}$
	C	1	$\frac{3}{2}$	$\frac{5}{2}$
	E	1	$\frac{3}{2}$	$\frac{5}{2}$
	F	1	$\frac{5}{2}$	$\frac{7}{2}$

3. Absorptive corrections

All single particle exchange models suffer from the fact that the low partial waves contribute too strongly to the final amplitude, thus leading to violation of unitarity. It has been customary when dealing with Regge models to appeal to Regge cuts, or absorption to rectify this. This has led to good results for Regge models (Adjei *et al* 1971a). In this paper we propose to apply the same procedure, that is, we regard the Veneziano amplitude as a Born term, which in view of its reduction to the Regge formula at asymptotic energies seems a reasonable assumption. This approach has been proposed by Lovelace and Ammann in a slightly different context. The absorption corrections are incorporated by first partial wave analysing the independent helicity amplitudes

$$\langle\lambda_3\lambda_4|T^J(s)|\lambda_1\lambda_2\rangle = \tfrac{1}{2}\int_{-1}^{1} \mathrm{d}(\cos\theta)\mathrm{d}^J_{\lambda\mu}(\theta)\langle\lambda_3\lambda_4|T|\lambda_1\lambda_2\rangle$$

where

$$\lambda = \lambda_1-\lambda_2 \qquad \mu = \lambda_3-\lambda_4.$$

We then modify this according to the Watson prescription, giving the modified partial wave amplitude

$$\langle\lambda_3\lambda_4|T'^J(s)|\lambda_1\lambda_2\rangle = \langle\lambda_3\lambda_4|S^J_{\mathrm{el}}|\lambda_3\lambda_4\rangle\langle\lambda_3\lambda_4|T^J(s)|\lambda_1\lambda_2\rangle$$

where we assume that S^J_{el} is pure nonflip and that the final state elastic scattering is the same as that of the initial state. S^J_{el} is parametrized by a real Gaussian and in the usual notation is

$$S^J_{\mathrm{el}} = 1-c\frac{\exp\{-J(J+1)\}}{\nu^2q^2}$$

where c and ν are fixed from elastic scattering data as shown in Adjei *et al* (1971a). The values of c and ν at the different values of s are given in table 4.

Table 4. Absorption coefficients

P_{lab}(GeV/c)	ν^{-1}(GeV)	c
1·96	0·40	1·0
5·0	0·32	0·71
7·3	0·29	0·58
12·7	0·27	0·51

The introduction of absorption destroys the crossing symmetry, but the leading order cut is 'duality preserving' (Krzywicki 1970). This is because the diffractive effect is equivalent to a fixed pole pomeron.

4. Trajectories

Since we have chosen to use a Veneziano formalism we are forced to use trajectories which have a common slope. We have chosen this to be 0·95 in common with previous work done by this group. This is obviously unrealistic for the exchange degenerate $\pi - \mathrm{B}$ trajectory, but we constrain this straight line trajectory to pass through the pion pole position on the Chew–Frautschi plot. The trajectories used are, of course, straight line trajectories with intercepts given in table 5.

Table 5. Intercepts of Regge trajectories

Pole	Intercept
π, B	−0·019
ρ, A_2	0·44
ω, f	0·417
ϕ, f′	0·014
η	−0·286
Y_0^*	−0·67
Y_1^*	−0·33

5. Discussion of results

We have compared the predictions of our model with the differential cross sections in the near-forward region and with the density matrices. The results for $K^+p \rightarrow K^0\Delta^{++}$ are shown in figures 1 and 2 and for $K^+p \rightarrow K^{*+}p$ in figures 3 and 4.

For $K^0\Delta^{++}$ the results are essentially the same as for the U(6, 6) Regge model, so we have not included those results in figures 1 and 2. The t dependence of the differential cross section is reproduced adequately, but the normalization is too low by almost an order of magnitude. This is due to the poor prediction of the $\bar{N}\Delta$ coupling given by the U(6, 6) scheme. The s dependence is correct even to the lowest energy. This is to be expected as the amplitude has been constructed with Regge asymptotic behaviour. This is also reflected in the good fit to the density matrix.

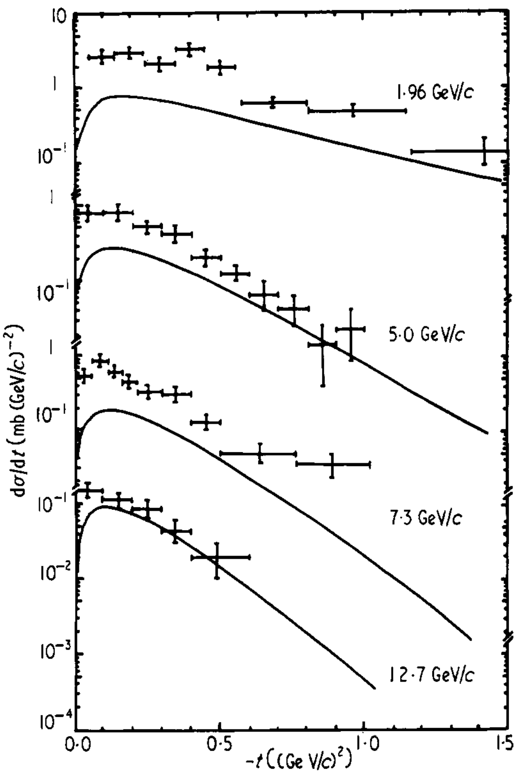

Figure 1. Differential cross section for $K^+p \to K^0\Delta^{++}$.

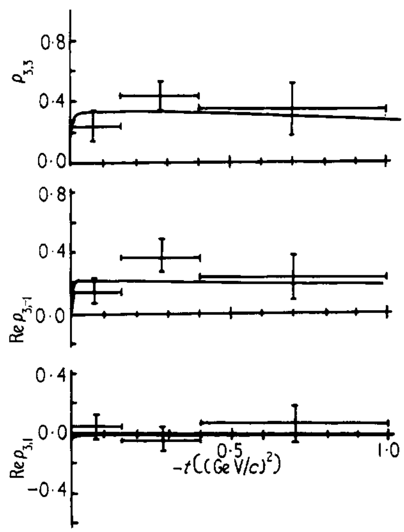

Figure 2. Density matrices of Δ^{++} at 7·3 GeV/c.

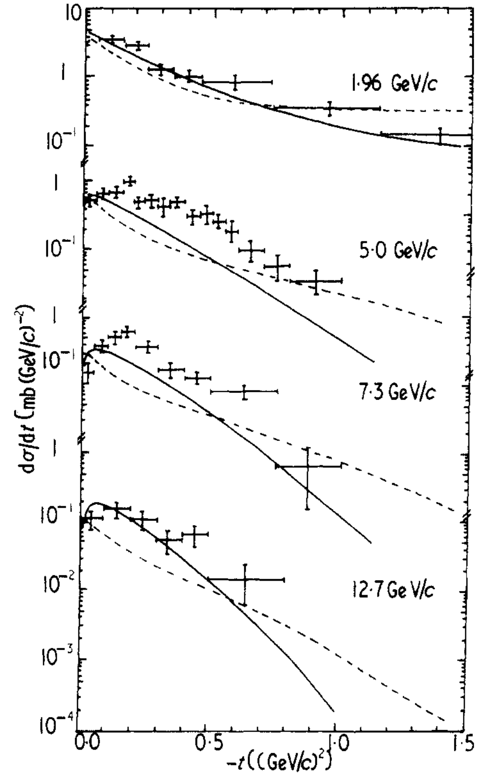

Figure 3. Differential cross section for $K^+p \rightarrow K^{*+}p$. The broken curves are the results of the Regge calculation of Adjei *et al* (1971a).

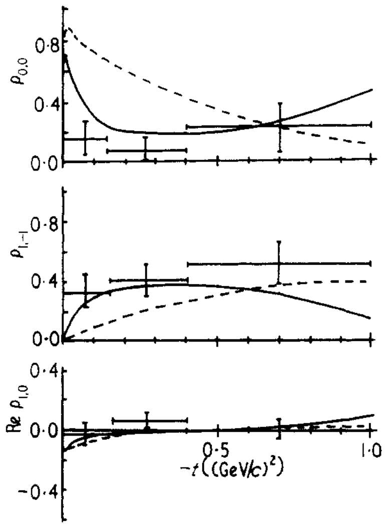

Figure 4. Density matrices of K^{*+} at 7·3 GeV/c. The broken curves are the results of the Regge calculation of Adjei *et al* (1971a).

For $K^{*+}p$ the results are an improvement over the U(6, 6) Regge model (Adjei *et al* 1971a) which are shown in figures 3 and 4 in that we have the correct s dependence, and the t dependence is much improved. However, the peak in $d\sigma/dt$ is shifted too much towards $t = 0$. This is presumably due to the failure of the symmetry scheme to predict the correct ratio of the pseudoscalar and vector Regge poles. The density matrices are also in much better agreement. The improvements are partly due to the fact that we have used a steeper pion trajectory, partly due to the inclusion of the backward poles, and partly due to a different choice of scale factor s_0, which in our case is chosen to be the reciprocal of the common slope of the trajectories.

While our amplitudes are constructed to give the correct asymptotic energy behaviour in the u channel, we have not attempted to discuss the crossed reactions, as there are little data available at present. It has been shown by Chan *et al* (1970), when baryon lines are crossed the predicted total cross section is much too large. Consequently we would not expect to get good values for the normalization of the $d\sigma/du$, although we would expect the correct energy dependence.

The results were found not to be very sensitive to the parameter P introduced in § 2. The extreme cases where $P = 0$ and 1 differed only by about 10%. We have therefore shown the results only for $P = 1$.

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